FYJC - MATHEMATICS & STATISTICS

PAPER - I

PLANE COORDINATE GEOMETRY

LOCUS

EXERCISE - 6.1

OTHER COLLEGES

Compiled & Conducted @ JKSC

EXERCISE - 6.1 - PART I

01. find the equation of the locus of a point whose distance from point (2, –3) is always 5 units . Also find where it cuts X – axis

ans : $x^2 + y^2 - 4x + 6y - 12 = 0$, cuts x – axis at (6,0) & (–2,0)

02. find the equation of the locus of a point whose distance from point (1,2) is always 5 units . Check whether the point (4,6) lies on the locus

ans : $x^2 + y^2 - 2x - 4y - 20 = 0$, (4,6) lies on the locus

03. find the equation of the locus of the point which is equidistant from the points A(5, –2) and B(–4,2) .

ans : $18x - 8y - 9 = 0$

04. find the equation of the locus of the point which is equidistant from the points A(3, 2) and (–5,6) . Check whether the points (1,8) and (–2,5) belong to the locus

ans : $2x - y + 6 = 0$, (1,8) lies on the locus ; (–2,5) does not lie on the locus

05. A(3,5) and B(4,1) . Find the locus of point P such that $l(AP)^2 + l(BP)^2 = 60$

ans : $2x^2 + 2y^2 - 14x - 12y - 9 = 0$

06. A(3,4) , B(–5,2) , C(2,1) . Find equation of locus of point P such that $l(AP)^2 + l(BP)^2 = l(CP)^2$

ans : $x^2 + y^2 + 8x - 10y + 49 = 0$

07. find the equation of the locus of the point P such that join of (–2,3) and (6, –7) subtends right angle at P

ans : $x^2 + y^2 - 4x + 4y - 33 = 0$

08. Find the equation of the locus of point P such that sum of squares of its distances from the points (3,0) and (0, –4) is 12

ans : $2x^2 + 2y^2 - 6x + 8y + 13 = 0$

09. A(2,4) and B(5,8) . Find the equation of locus of point P such that $l(AP)^2 - l(BP)^2 = 13$

ans : $3x + 4y - 41 = 0$

10. find the equation of the locus of the point whose distance from $(-2,1)$ is thrice its distance from $(1,4)$

ans : $4x^2 + 4y^2 - 11x - 35y + 74 = 0$

11. Find the equation of locus of point P such that 4(AP) = 3(BP) where A(−4, −2) and B(−1,1)

ans : $7x^2 + 7y^2 + 110x + 82y + 302 = 0$

12. if A (4,0) and B (−4,0) are two points , show that equation of the locus of a point P such that $l(AP) + l(BP) = 10$ is $\mathbf{\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1}$

13. if A (0,2) and B (0, −2) are two points , show that equation of the locus of a point P, such that PA + PB = 6 is given by $\mathbf{\dfrac{x^2}{5} + \dfrac{y^2}{9} = 1}$

14. if A (−15,0) and B (15,0) are two points , show that equation of the locus of a point P such that $l(AP) - l(BP) = 24$ is $\mathbf{\dfrac{x^2}{144} - \dfrac{y^2}{81} = 1}$

15. A point P moves such that the sum of the distances from the points (c,0) and (−c,0) is 2a . Show that equation of its locus is $\mathbf{\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1}$ **, where $b^2 = a^2 - c^2$**

QUESTIONS LIKELY FOR MCQ'S

Q1: Find locus of a point P such that

a) abscissa is three times the ordinate **ans :** $x = 3y$

b) abscissa exceeds twice its ordinate by 7 **ans :** $x = 2y + 7$

c) ordinate of P exceeds three times its abscissa by 5 **ans :** $y = 3x + 5$

d) distance from y – axis is three times its distance from the origin **ans :** $8x^2 + 9y^2 = 0$

e) its distance from the origin is three times its distance from the x – axis **ans :** $x^2 - 8y^2 = 0$

Q2:

01. Find k if the point (−8,6) lies on the locus $\dfrac{x^2}{4} + \dfrac{y^2}{3} = k$ **ans :** $k = 28$

02. Find the value of k if point $(-2,2)$ lies on the locus $x^2 - 7x + ky = 0$. If the point $Q(3,a)$ also lies on the locus , find the value of 'a' **ans :** $k = -9$; $a = -4/3$

03. Find the values of a and b if the points $(3,2)$ and $(-1, -2)$ lie on the locus $ax + by = 5$ **ans :** $a = 5$, $b = -5$

SOLUTION TO EXERCISE - 6.1 - PART - I

01. find the equation of the locus of a point whose distance from point (2, –3) is always 5 units . Also find where it cuts X – axis

SOLUTION : let P(x,y) be any point on the locus , A((2, –3)

As per the given condition

$PA = 5$

$PA^2 = 25$

$(x - 2)^2 + (y + 3)^2 = 25$

$x^2 - 4x + 4 + y^2 + 6y + 9 = 25$

$x^2 + y^2 - 4x + 6y + 13 - 25 = 0$

$x^2 + y^2 - 4x + 6y - 12 = 0$ ……. Equation of the locus

Put $y = 0$

$x^2 - 4x - 12 = 0$

$x^2 - 6x + 2x - 12 = 0$

$(x - 6)(x + 2) = 0$

$x = 6$, $x = -2$

Hence cuts x – axis at (6,0) & (–2,0)

02. find the equation of the locus of a point whose distance from point (1,2) is always 5 units . Check whether the point (4,6) lies on the locus

SOLUTION : a) let P(x,y) be any point on the locus , A(1, 2)

As per the given condition

$PA = 5$

$PA^2 = 25$

$(x - 1)^2 + (y - 2)^2 = 25$

$x^2 - 2x + 1 + y^2 - 4y + 4 = 25$

$x^2 + y^2 - 2x - 4y + 5 - 25 = 0$

$x^2 + y^2 - 2x - 4y - 20 = 0$ ……. Equation of the locus

b) subs (4,6) in the equation of the locus

$4^2 + 6^2 - 2(4) - 4(6) - 20 = 0$

$16 + 36 - 8 - 24 - 20 = 0$

$52 - 52 = 0$

$0 = 0$

(4,6) satisfies the equation of the locus and hence it lies on the locus

03. find the equation of the locus of the point which is equidistant from the points A(5, −2) and B(−4,2) .

SOLUTION : let P(x,y) be any point on the locus , A(5, −2) ; B(−4,2)

As per the given condition

$PA = PB$

$PA^2 = PB^2$

$(x-5)^2 + (y+2)^2 = (x+4)^2 + (y-2)^2$

$\cancel{x^2} - 10x + 25 + \cancel{y^2} + 4y + 4 = \cancel{x^2} + 8x + 16 + \cancel{y^2} - 4y + 4$

$-10x + 4y + 29 = 8x - 4y + 20$

$0 = 18x - 8y - 9$

$18x - 8y - 9 = 0$ ………… Equation of the locus

04. find the equation of the locus of the point which is equidistant from the points A(3, 2) and (−5,6) . Check whether the points (1,8) and (−2,5) belong to the locus

SOLUTION : a) let P(x,y) be any point on the locus , A(3, 2) ; B−5,6)

As per the given condition

$PA = PB$

$PA^2 = PB^2$

$(x-3)^2 + (y-2)^2 = (x+5)^2 + (y-6)^2$

$\cancel{x^2} - 6x + 9 + \cancel{y^2} - 4y + 4 = \cancel{x^2} + 10x + 25 + \cancel{y^2} - 12y + 36$

$-6x - 4y + 13 = 10x - 12y + 61$

$0 = 16x - 8y + 48$

$\div 8$

$2x - y + 6 = 0$ ………… Equation of the locus

b) Put (1,8) in the equation of the locus

$2 - 8 + 6 = 0$

$0 = 0$

(1,8) satisfies the equation of the locus and hence it lies on the locus

c) Put (−2,5) in the equation of the locus

$2(-2) - 5 + 6 = 0$

$-4 - 5 + 6 = 0$

$-3 \neq 0$

(−2,5) does not satisfy the equation of the locus and hence it does not lie on the locus

05. A(3,5) and B(4,1) . Find the locus of point P such that $l(AP)^2 + l(BP)^2 = 60$

SOLUTION : let P(x,y) be any point on the locus , A(3, 5) ; B(4,1)

As per the given condition

$l(AP)^2 + l(BP)^2 = 60$

$(x-3)^2 + (y-5)^2 + (x-4)^2 + (y-1)^2 = 60$

$$\begin{array}{rl} & x^2 - 6x + 9 + y^2 - 10y + 25 \\ + & x^2 - 8x + 16 + y^2 - 2y + 1 \quad = 60 \\ \hline & 2x^2 - 14x + 25 + 2y^2 - 12y + 26 \quad = 60 \end{array}$$

$2x^2 + 2y^2 - 14x - 12y + 51 - 60 = 0$

$2x^2 + 2y^2 - 14x - 12y - 9 = 0$ ……. Locus of P

06. A(3,4) , B(–5,2) , C(2,1) . Find equation of locus of point P such that $l(AP)^2 + l(BP)^2 = l(CP)^2$

SOLUTION : let P(x,y) be any point on the locus , A(3, 5) ; B(4,1)

As per the given condition

$l(AP)^2 + l(BP)^2 = l(CP)^2$

$(x-3)^2 + (y-4)^2 + (x+5)^2 + (y-2)^2 = (x-2)^2 + (y-1)^2$

$$\begin{array}{rll} & x^2 - 6x + 9 + y^2 - 8y + 16 & \\ + & x^2 + 10x + 25 + y^2 - 4y + 4 & = x^2 - 4x + 4 + y^2 - 2y + 1 \\ \hline & 2x^2 + 4x + 34 + 2y^2 - 12y + 20 & = x^2 - 4x + 4 + y^2 - 2y + 1 \end{array}$$

$2x^2 + 2y^2 + 4x - 12y + 54 = x^2 + y^2 - 4x - 2y + 5$

$x^2 + y^2 + 8x - 10y + 49 = 0$ ……. Locus of P

07. find the equation of the locus of the point P such that join of (–2,3) and (6, –7) subtends right angle at P

SOLUTION : a) let P(x,y) be any point on the locus , A(–2,3) ; B(6,–7)

$PA^2 + PB^2 = AB^2$

$\left[(x+2)^2 + (y-3)^2\right] + \left[(x-6)^2 + (y+7)^2\right] = (-2-6)^2 + (3+7)^2$

$x^2 + 4x + 4 + y^2 - 6y + 9 + x^2 - 12x + 36 + y^2 + 14y + 49 = 64 + 100$

$2x^2 + 2y^2 - 8x + 8y + 13 + 85 = 164$

$2x^2 + 2y^2 - 8x + 8y + 98 - 164 = 0$

$2x^2 + 2y^2 - 8x + 8y - 66 = 0$

$x^2 + y^2 - 4x + 4y - 33 = 0$ ………. equation of the locus

08. Find the equation of the locus of point P such that sum of squares of its distances from the points (3,0) and (0, −4) is 12

SOLUTION : let P(x,y) be any point on the locus , A(3,0) ; B(0,−4)

As per the given condition

$PA^2 + PB^2 = 12$

$\left[(x-3)^2 + (y-0)^2\right] + \left[(x-0)^2 + (y+4)^2\right] = 12$

$x^2 - 6x + 9 + y^2 + x^2 + y^2 + 8y + 16 = 12$

$2x^2 + 2y^2 - 6x + 8y + 25 - 12 = 0$

$2x^2 + 2y^2 - 6x + 8y + 13 = 0$ ……… equation of the locus

09. A(2,4) and B(5,8) . Find the equation of locus of point P such that $l(AP)^2 - l(BP)^2 = 13$

SOLUTION : let P(x,y) be any point on the locus , A(2,4) ; B(5,8)

As per the given condition

$AP^2 - BP^2 = 13$

$\left[(x-2)^2 + (y-4)^2\right] - \left[(x-5)^2 + (y-8)^2\right] = 13$

$\left[x^2 - 4x + 4 + y^2 - 8y + 16\right] - \left[x^2 - 10x + 25 + y^2 - 16y + 64\right] = 13$

$x^2 - 4x + 4 + y^2 - 8y + 16 - x^2 + 10x - 25 - y^2 + 16y - 64 = 13$

$6x + 8y + 20 - 89 = 13$

$6x + 8y - 69 - 13 = 0$

$6x + 8y - 82 = 0$

$3x + 4y - 41 = 0$ ……… equation of the locus

10. find the equation of the locus of the point whose distance from (–2,1) is thrice its distance from (1,4)

SOLUTION : a) let P(x,y) be any point on the locus , A(–2,1) ; B(1,4)

As per the given condition

$PA = 3PB$

$PA^2 = 9PB^2$

$(x + 2)^2 + (y - 1)^2 = 9\left[(x - 1)^2 + (y - 4)^2\right]$

$x^2 + 4x + 4 + y^2 - 2y + 1 = 9(x^2 - 2x + 1 + y^2 - 8y + 16)$

$x^2 + y^2 + 4x - 2y + 5 = 9(x^2 + y^2 - 2x - 8y + 17)$

$x^2 + y^2 + 4x - 2y + 5 = 9x^2 + 9y^2 - 18x - 72y + 153$

$0 = 8x^2 + 8y^2 - 22x - 70y + 148$

$8x^2 + 8y^2 - 22x - 70y + 148 = 0$

$\div 2$

$4x^2 + 4y^2 - 11x - 35y + 74 = 0$ ……… equation of the locus

11. Find the equation of locus of point P such that 4(AP) = 3(BP) where A(–4, –2) and B(–1,1)

SOLUTION : let P(x,y) be any point on the locus , A(–4, –2) ; B(–1,1)

As per the given condition

$4(AP) = 3(BP)$

$16AP^2 = 9BP^2$

$16\left[(x + 4)^2 + (y + 2)^2\right] = 9\left[(x + 1)^2 + (y - 1)^2\right]$

$16\left[x^2 + 8x + 16 + y^2 + 4y + 4\right] = 9\left[x^2 + 2x + 1 + y^2 - 2y + 1\right]$

$16\left[x^2 + y^2 + 8x + 4y + 20\right] = 9\left[x^2 + y^2 + 2x - 2y + 2\right]$

$16x^2 + 16y^2 + 128x + 64y + 320 = 9x^2 + 9y^2 + 18x - 18y + 18$

$7x^2 + 7y^2 + 110x + 82y + 302 = 0$ ………. equation of the locus

12. if A (4,0) and B (–4,0) are two points , show that equation of the locus of a point P such that $l(AP) + l(BP) = 10$ is $\mathbf{\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1}$

SOLUTION : let P(x,y) be any point on the locus , A(4,0) ; B(–4,0)

As per the given condition

$AP + BP = 10$

$AP = 10 - BP$

$AP^2 = (10 - BP)^2$

$AP^2 = 100 - 20BP + BP^2$

$(x - 4)^2 + y^2 = 100 - 20BP + (x + 4)^2 + y^2$

$\cancel{x^2} - 8x + \cancel{16} + \cancel{y^2} = 100 - 20BP + \cancel{x^2} + 8x + \cancel{16} + \cancel{y^2}$

$20BP = 100 + 16x$

Dividing throughout by 4

$5BP = 25 + 4x$

Squaring on both sides

$25BP^2 = (25 + 4x)^2$

$25\left[(x + 4)^2 + y^2\right] = 625 + 200x + 16x^2$

$25\left[x^2 + 8x + 16 + y^2\right] = 625 + 200x + 16x^2$

$25x^2 + 200x + 400 + 25y^2 = 625 + 200x + 16x^2$

$9x^2 + 25y^2 = 225$

Dividing throughout by 225

$$\frac{9x^2}{225} + \frac{25y^2}{225} = \frac{225}{225}$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

locus traced by P ...... proved

13. if A (0,2) and B (0, –2) are two points , show that equation of the locus of a point P, such that PA + PB = 6 is given by

$$\frac{x^2}{5} + \frac{y^2}{9} = 1$$

SOLUTION : let P(x,y) be any point on the locus , A(0,2) ; B(0,–2)

As per the given condition

$PA + PB = 6$

$PA = 6 - PB$

$PA^2 = (6 - PB)^2$

$PA^2 = 36 - 12PB + PB^2$

$x^2 + (y - 2)^2 = 36 - 12PB + x^2 + (y + 2)^2$

$\cancel{x^2} + \cancel{y^2} - 4y + \cancel{4} = 36 - 12PB + \cancel{x^2} + \cancel{y^2} + 4y + \cancel{4}$

$12PB = 36 + 8y$

Dividing throughout by 4

$3PB = 9 + 2y$

Squaring on both sides

$9PB^2 = (9 + 2y)^2$

$9\left[x^2 + (y + 2)^2\right] = 81 + 36y + 4y^2$

$9\left[x^2 + y^2 + 4y + 4\right] = 81 + 36y + 4y^2$

$9x^2 + 9y^2 + 36y + 36 = 81 + 36y + 4y^2$

$9x^2 + 5y^2 = 45$

Dividing throughout by 45

$$\frac{9x^2}{45} + \frac{5y^2}{45} = \frac{45}{45}$$

$$\frac{x^2}{5} + \frac{y^2}{9} = 1$$

locus traced by P …… proved

14. if A $(-15,0)$ and B $(15,0)$ are two points , show that equation of the locus of a point P such that $l(AP) - l(BP) = 24$ is $\mathbf{\dfrac{x^2}{144} - \dfrac{y^2}{81} = 1}$

SOLUTION : let P(x,y) be any point on the locus , A(15,0) ; B(−15,0)

As per the given condition

$AP - BP = 24$

$AP = 24 + BP$

$AP^2 = (24 + BP)^2$

$AP^2 = 576 + 48BP + BP^2$

$(x + 15)^2 + y^2 = 576 + 48BP + (x - 15)^2 + y^2$

$\cancel{x^2} + 30x + \cancel{225} + \cancel{y^2} = 576 + 48BP + \cancel{x^2} - 30x + \cancel{225} + \cancel{y^2}$

$48BP = 60x - 576$

Dividing throughout by 12

$4BP = 5x - 48$

Squaring on both sides

$16BP^2 = (5x - 48)^2$

$16\left[(x - 15)^2 + y^2\right] = 25x^2 - 480x + 2304$

$16\left[x^2 - 30x + 225 + y^2\right] = 25x^2 - 480x + 2304$

$16x^2 - \cancel{480x} + 3600 + 16y^2 = 25x^2 - \cancel{480x} + 2304$

$3600 - 2304 = 9x^2 - 16y^2$

$9x^2 - 16y^2 = 1296$

Dividing throughout by 1296

$$\frac{9x^2}{1296} - \frac{16y^2}{1296} = 1$$

$$\frac{x^2}{144} - \frac{y^2}{81} = 1$$

locus traced by P ...... proved

15. A point P moves such that the sum of the distances from the points (c,0) and (–c,0) is 2a . Show that equation of its locus is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ , where } b^2 = a^2 - c^2 .$$

SOLUTION : let P(x,y) be any point on the locus , A(c,0) ; B(–c,0)

As per the given condition

$AP + BP = 2a$

$AP = 2a - BP$

$AP^2 = (2a - BP)^2$

$AP^2 = 4a^2 - 4a.BP + BP^2$

$(x - c)^2 + y^2 = 4a^2 - 4a.BP + (x + c)^2 + y^2$

$x^2 - 2xc + c^2 + y^2 = 4a^2 - 4a.BP + x^2 + 2xc + c^2 + y^2$

$4a.BP = 4a^2 + 4xc$

Dividing throughout by 4

$a.BP = a^2 + xc$

Squaring on both sides

$a^2BP^2 = (a^2 + xc)^2$

$a^2\left[(x + c)^2 + y^2\right] = a^4 + 2a^2xc + x^2c^2$

$a^2\left[x^2 + 2xc + c^2 + y^2\right] = a^4 + 2a^2xc + x^2c^2$

$a^2x^2 + 2a^2xc + a^2c^2 + a^2y^2 = a^4 + 2a^2xc + x^2c^2$

$a^2x^2 - x^2c^2 + a^2y^2 = a^4 - a^2c^2$

$x^2(a^2 - c^2) + a^2y^2 = a^2(a^2 - c^2)$

$x^2b^2 + a^2y^2 = a^2b^2$

Dividing throughout by a^2b^2

$$\frac{x^2b^2}{a^2b^2} + \frac{a^2y^2}{a^2b^2} = \frac{a^2b^2}{a^2b^2}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

locus traced by P ...... proved

QUESTIONS LIKELY FOR MCQ'S

Q1: Find locus of a point P such that

a) abscissa is three times the ordinate

let P(x,y) be any point on the locus

as per the given condition : $x = 3y$ …… **equation of the locus**

b) abscissa exceeds twice its ordinate by 7

let P(x,y) be any point on the locus

as per the given condition : $x - 2y = 7$ ….. equation of the locus

c) ordinate of P exceeds three times its abscissa by 5

let P(x,y) be any point on the locus

as per the given condition : $y - 3x = 5$ ….. equation of the locus

d) distance from y – axis is three times its distance from the origin

let P(x,y) be any point on the locus

as per the given condition : $x = 3OP$

$x^2 = 9OP^2$

$x^2 = 9(x^2 + y^2)$

$x^2 = 9x^2 + 9y^2$

$8x^2 + 9y^2 = 0$ ….. equation of the locus

e) its distance from the origin is three times its distance from the x – axis **ans :** $x^2 - 8y^2 = 0$

let P(x,y) be any point on the locus

as per the given condition : $OP = 3y$

$OP^2 = 9y^2$

$x^2 + y^2 = 9y^2$

$x^2 - 8y^2 = 0$ ….. equation of the locus

Q2:

01. Find k if the point (–8,6) lies on the locus $\frac{x^2}{4} + \frac{y^2}{3} = k$

ans : k = 28

Since (–8,6) lies on locus $\frac{x^2}{4} + \frac{y^2}{3} = k$;

it must satisfy the equation , hence sub (–8,6) $\frac{64}{4} + \frac{36}{3} = k$

$16 + 12 = k \qquad k = 28$

02. Find the value of k if point (–2,2) lies on the locus $x^2 - 7x + ky = 0$. If the point Q(3,a) also lies on the locus , find the value of 'a'

point (–2,2) lies on the locus $x^2 - 7x + ky = 0$

subs : $(-2)^2 - 7(-2) + k(2) = 0$

$4 + 14 + 2k = 0$

$18 + 2k = 0$

$2k = -18$

$k = -9$

Hence equation of the locus : $x^2 - 7x - 9y = 0$

Now ,

Q(3,a) lies on locus $x^2 - 7x - 9y = 0$

Subs $3^2 - 7(3) - 9a = 0$

$9 - 21 - 9a = 0$

$-12 - 9a = 0$

$-9a = 12$ $\qquad a = -4/3$

03. Find the values of a and b if the points (3,2) and (–1, –2) lie on the locus ax + by = 5

(3,2) and (–1, –2) lie on the locus ax + by = 5

subs

$$\begin{array}{rcl} 3a + 2b & = & 5 \\ -a - 2b & = & 5 \\ \hline 2a & = & 10 \\ a & = & 5 \end{array}$$

subs in (2) $-5 - 2b = 5$

$-2b = 10$

$b = -5$

EXERCISE - 6.1 - PART II

01. A is a point on X – axis and B is a point on Y – axis such that l(AB) = 7 . Find the locus of midpoint of segment AB

ans : $4x^2 + 4y^2 = 49$

02. A and B are variable points on the x and y axes respectively . If AB = 4 , find the locus of the point which divides seg AB internally in the ratio 1 : 2

ans : $9x^2 + 36y^2 = 64$

03. A is a point on X – axis and B is a point on Y axis such that l(AB) = 10 . Find the equation of the locus of a point P which divides segment AB externally in the ratio 2 : 1

ans : $4x^2 + y^2 = 400$

04. $A \equiv (3,5)$,B is any point on the locus whose equation is $x^2 + y^2 = 100$. Find the locus of midpoint of segment AB

ans : $2x^2 + 2y^2 - 6x - 10y - 33 = 0$

05. $A \equiv (-4,0)$ is a given point and B is any point on the locus whose equation is $x^2 - y^2 + 4 = 0$. Find the locus of point which divides seg AB internally in ratio 1 : 3

ans : $4x^2 - 4y^2 + 24x + 37 = 0$

06. $A \equiv (2,1)$ is a fixed point . A variable point B lies on the locus $x^2 + y^2 + 5x - 5 = 0$. Find the equation of the locus of point which divides seg AB internally in ratio 1 : 2

ans : $9x^2 + 9y^2 - 9x - 12y - 5 = 0$

07. $A \equiv (-3,0)$, P is any point on the locus whose equation is $y^2 = 8x$. If Q divides AP internally in the ratio 3 : 2, find the locus of Q

ans : $25y^2 = 24(5x + 6)$

08. Find the equation of the locus of point P which divides AQ externally in the ratio 2 : 1 where A (3, –4) and Q is a point on the locus $x^2 = 6y$

ans : $(x + 3)^2 = 12(y - 4)$

09. Q is a point on $x^2 + y^2 + 6x - 4y + 9 = 0$. Find the equation of locus of P which divides seg OQ externally in the ratio 2 : 5 , O being the origin

ans : $9x^2 + 9y^2 - 36x + 24y + 36 = 0$

10. A(2,5) and B(9,–14) are vertices of ΔABC . The third vertex C lies on the locus whose equation is $3x + 4y + 5 = 0$. Find the locus of the centroid of triangle ABC

ans : $9x + 12y + 8 = 0$

11. A(2, –5) and B(–7,6) are the two vertices of ΔABC . Find the equation of the locus of third vertex C , if the centroid of the triangle lies on the line $3x - 4y + 11 = 0$

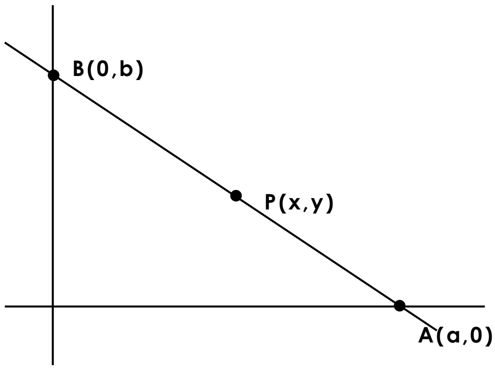
ans : $3x - 4y + 14 = 0$

12. The centroid of a triangle always lies on the locus $y = 5 + 2x^2$. If A (–5,2) & C(4, –3) , find the equation of the locus of third vertex B

ans : $3y = 2x^2 - 4x + 50$

01. A is a point on X – axis and B is a point on Y – axis such that l(AB) = 7 . Find the locus of midpoint of segment AB

SOLUTION : Let P(x,y) be any point on the locus



PART – 1

AB = 7

$AB^2 = 49$

$(a - 0)^2 + (0 - b)^2 = 49$

$a^2 + b^2 = 49$ ... (1)

PART – 2

P(x,y) is the midpoint of seg AB

Using Midpoint formula

$X = \dfrac{x_1 + x_2}{2}$	$y = \dfrac{y_1 + y_2}{2}$
$x = \dfrac{a + 0}{2}$	$y = \dfrac{0 + b}{2}$
$2x = a$	$2y = b$

Subs in (1)

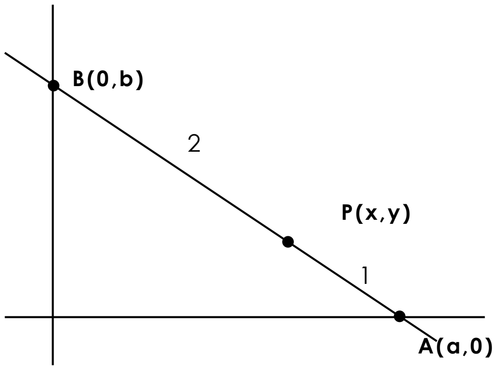
$(2x)^2 + (2y)^2 = 49$

$4x^2 + 4y^2 = 49$ ..... locus traced by P

02. A and B are variable points on the x and y axes respectively . If AB = 4 , find the locus of the point which divides seg AB internally in the ratio 1 : 2

SOLUTION

Let P(x,y) be any point on the locus



PART – 1

$AB = 4$

$AB^2 = 16$

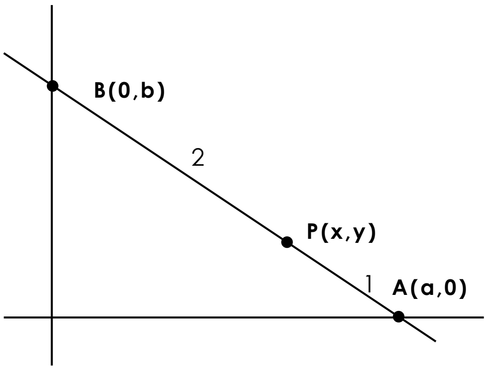
$(a - 0)^2 + (0 - b)^2 = 49$

$a^2 + b^2 = 16 \quad \ldots (1)$

PART – 2

P(x,y) divides seg AB internally in the ratio 1 : 2

Using section formula (internal division)



$X = \dfrac{mx_2 + nx_1}{m + n}$	$y = \dfrac{my_2 + ny_1}{m + n}$
$x = \dfrac{2(a) + 1(0)}{2 + 1}$	$y = \dfrac{2(0) + 1(b)}{2 + 1}$
$x = \dfrac{2a}{3}$	$y = \dfrac{b}{3}$
$a = \dfrac{3x}{2}$	$b = 3y$

Subs in (1)

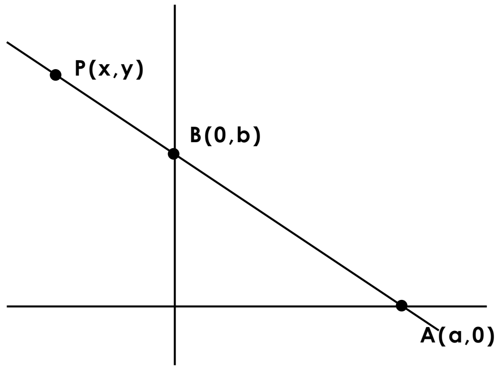
$$\left(\frac{3x}{2}\right)^2 + (3y)^2 = 16$$

$$\frac{9x^2}{4} + 9y^2 = 16$$

$$9x^2 + 36y^2 = 64 \quad \ldots\ldots \text{ locus traced by P}$$

03. A is a point on X – axis and B is a point on Y axis such that l(AB) = 10 . Find the equation of the locus of a point P which divides segment AB externally in the ratio 2 : 1

SOLUTION : Let P(x,y) be any point on the locus



PART – 1

$AB = 10$

$AB^2 = 100$

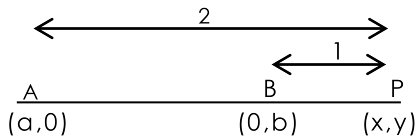
$(a - 0)^2 + (0 - b)^2 = 100$

$a^2 + b^2 = 100 \quad \ldots (1)$

PART – 2

P(x,y) divides seg AB externally in the ratio 2 : 1

Using section formula (external division)



$$x = \frac{mx_2 - nx_1}{m - n}$$

$$x = \frac{2(0) - 1(a)}{2 - 1}$$

$$x = \frac{0 - a}{1}$$

$$x = -a$$

$$a = -x$$

$$y = \frac{my_2 - ny_1}{m - n}$$

$$y = \frac{2(b) - 1(0)}{2 - 1}$$

$$y = \frac{2b - 0}{1}$$

$$y = 2b$$

$$b = \frac{y}{2}$$

Subs in (1)

$$(-x)^2 + \left(\frac{y}{2}\right)^2 = 100$$

$$x^2 + \frac{y^2}{4} = 100$$

$4x^2 + y^2 = 400$ ……… Locus traced by P

04. A ≡ (3,5) ,B is any point on the locus whose equation is $x^2 + y^2 = 100$. Find the locus of midpoint of segment AB

SOLUTION :

Let P(x,y) be any point on the required locus

PART – 1

Since B(a,b) is any point on the locus $x^2 + y^2 = 100$, it must satisfy the eq.

$$a^2 + b^2 = 100 \quad \ldots\ldots\ldots \quad (1)$$

PART – 2

P(x,y) is the midpoint of seg AB

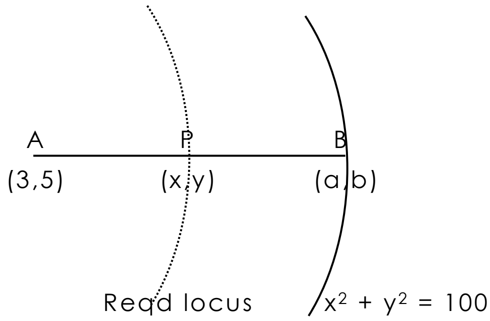
Using Midpoint formula

$x = \dfrac{x_1 + x_2}{2}$ $\qquad$ $y = \dfrac{y_1 + y_2}{2}$

$x = \dfrac{3 + a}{2}$ $\qquad$ $y = \dfrac{5 + b}{2}$

$2x = 3 + a$ $\qquad$ $2y = 5 + b$

$a = 2x - 3$ $\qquad$ $b = 2y - 5$



Subs in (1)

$(2x - 3)^2 + (2y - 5)^2 = 100$

$4x^2 - 12x + 9 + 4y^2 - 20y + 25 = 100$

$4x^2 + 4y^2 - 12x - 20y + 34 - 100 = 0$

$4x^2 + 4y^2 - 12x - 20y - 66 = 0$

$2x^2 + 2y^2 - 6x - 10y - 33 = 0$ $\quad \ldots\ldots\ldots$ locus traced by P

05. A ≡ (−4,0) is a given point and B is any point on the locus whose equation is $x^2 - y^2 + 4 = 0$.

Find the locus of point which divides seg AB internally in ratio 1 : 3

SOLUTION :

Let P(x,y) be any point on the required locus

PART – 1

Since B(a,b) is any point on the locus $x^2 - y^2 + 4 = 0$, it must satisfy the eq.

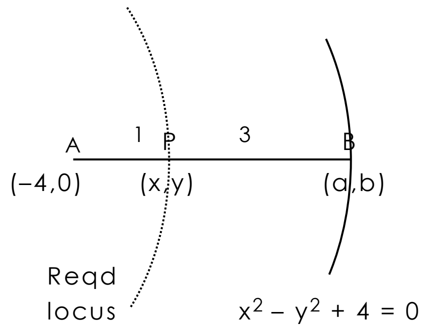
$$a^2 - b^2 + 4 = 0 \;\ldots\ldots\ldots \quad (1)$$

PART – 2

P(x,y) divides seg AB internally in the ratio 1 : 3

Using section formula (internal division)

$X = \dfrac{mx_2 + nx_1}{m + n}$	$y = \dfrac{my_2 + ny_1}{m + n}$
$x = \dfrac{1(a) + 3(-4)}{1 + 3}$	$y = \dfrac{1(b) + 3(0)}{1 + 3}$
$x = \dfrac{a - 12}{4}$	$y = \dfrac{b}{4}$
$a = 4x + 12$	$b = 4y$



subs in (1)

$(4x + 12)^2 - (4y)^2 + 4 = 0$

$16x^2 + 96x + 144 - 16y^2 + 4 = 0$

$16x^2 - 16y^2 + 96x + 148 = 0$

$4x^2 - 4y^2 + 24x + 37 = 0$ ……… locus traced by P

06. A ≡ (2,1) is a fixed point . A variable point B lies on the locus $x^2 + y^2 + 5x - 5 = 0$. Find the equation of the locus of point which divides seg AB internally in ratio 1 : 2

SOLUTION :

Let P(x,y) be any point on the required locus

PART – 1

Since B(a,b) is any point on the locus $x^2 + y^2 + 5x - 5 = 0$, it must satisfy the equation

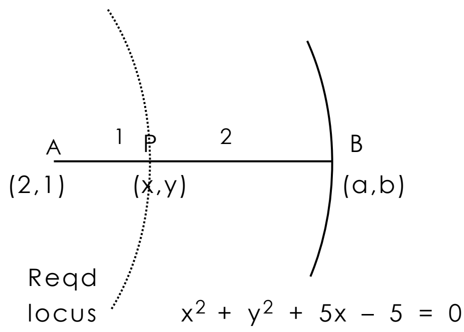
$$a^2 + b^2 + 5a - 5 = 0 \quad \ldots\ldots\ldots \quad (1)$$

PART – 2

P(x,y) divides seg AB internally in the ratio 1 : 2

Using section formula (internal division)

$X = \dfrac{mx_2 + nx_1}{m + n}$	$y = \dfrac{my_2 + ny_1}{m + n}$
$x = \dfrac{1(a) + 2(2)}{1 + 2}$	$y = \dfrac{1(b) + 2(1)}{1 + 2}$
$x = \dfrac{a + 4}{3}$	$y = \dfrac{b + 2}{3}$
$a = 3x - 4$	$b = 3y - 2$



subs in (1)

$(3x - 4)^2 + (3y - 2)^2 + 5(3x - 4) - 5 = 0$

$9x^2 - 24x + 16 + 9y^2 - 12y + 4 + 15x - 20 - 5 = 0$

$9x^2 + 9y^2 - 9x - 12y - 5 = 0$ ……… locus traced by P

07. $A \equiv (-3,0)$, P is any point on the locus whose equation is $y^2 = 8x$. If Q divides AP internally in the ratio 3 : 2, find the locus of Q

SOLUTION : Let Q(x,y) be any point on the required locus

PART – 1

Since P(a,b) is any point on the locus $y^2 = 8x$, it must satisfy the eq.

$$b^2 = 8a \quad \ldots\ldots\ldots \quad (1)$$

PART – 2

Q(x,y) divides seg AP internally in the ratio 3 : 2

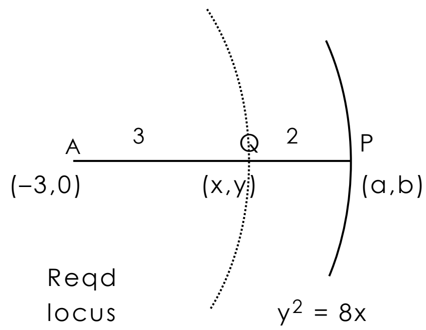
Using section formula (internal division)

$$x = \frac{mx_2 + nx_1}{m + n} \qquad y = \frac{my_2 + ny_1}{m + n}$$

$$x = \frac{3(a) + 2(-3)}{3 + 2} \qquad y = \frac{3(b) + 2(0)}{3 + 2}$$

$$x = \frac{3a - 6}{5} \qquad y = \frac{3b}{5}$$

$$a = \frac{5x + 6}{3} \qquad b = \frac{5y}{3}$$



subs in (1)

$$\left(\frac{5y}{3}\right)^2 = \frac{8(5x + 6)}{3}$$

$$\frac{25y^2}{9} = \frac{8(5x + 6)}{3}$$

$$25y^2 = 24(5x + 6) \quad \ldots\ldots\ldots \quad \text{locus traced by Q}$$

08. Find the equation of the locus of point P which divides AQ externally in the ratio 2 : 1 where A (3, –4) and Q is a point on the locus $x^2 = 6y$

SOLUTION : Let P(x,y) be any point on the required locus

PART – 1

Since Q(a,b) is any point on the locus $x^2 = 6y$, it must satisfy the eq.

$$a^2 = 6b \quad \ldots\ldots\ldots \quad (1)$$

PART – 2

P(x,y) divides seg AQ externally in the ratio 2 : 1 (PA:PQ = 2:1)

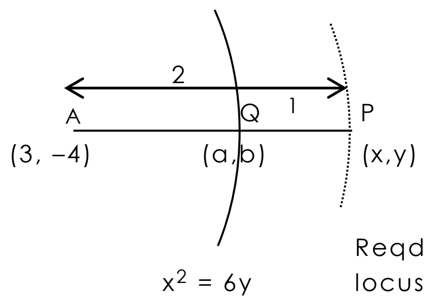
Using section formula (external division)

$$x = \frac{mx_2 - nx_1}{m - n} \qquad y = \frac{my_2 - ny_1}{m - n}$$

$$x = \frac{2(a) - 1(3)}{2 - 1} \qquad y = \frac{2(b) - 1(-4)}{2 - 1}$$

$$x = \frac{2a - 3}{1} \qquad y = \frac{2b + 4}{1}$$

$$a = \frac{x + 3}{2} \qquad b = \frac{y - 4}{2}$$



subs in (1)

$$\left(\frac{x + 3}{2}\right)^2 = 6\left(\frac{y - 4}{2}\right)$$

$$\frac{(x + 3)^2}{4} = 3(y - 4)$$

$$(x + 3)^2 = 12(y - 4)$$

09. Q is a point on $x^2 + y^2 + 6x - 4y + 9 = 0$. Find the equation of locus of P which divides seg OQ externally in the ratio 2 : 5 , O being the origin

SOLUTION : Let P(x,y) be any point on the required locus

PART – 1

Since Q(a,b) is any point on the locus $x^2 + y^2 + 6x - 4y + 9 = 0$, it must satisfy the equation

$$a^2 + b^2 + 6a - 4b + 9 = 0 \ldots\ldots\ldots \ (1)$$

PART – 2

P(x,y) divides seg OQ externally in the ratio 2 : 5 (PO:PQ = 2:5)

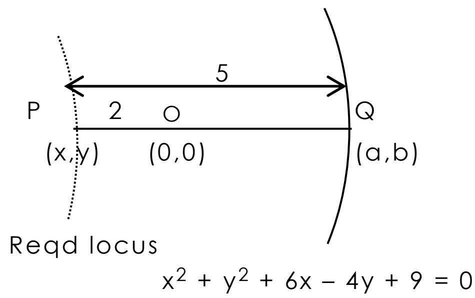
Using section formula (external division)

$$x = \frac{mx_2 - nx_1}{m - n} \qquad\qquad y = \frac{my_2 - ny_1}{m - n}$$

$$x = \frac{5(0) - 2(a)}{5 - 2} \qquad\qquad y = \frac{5(0) - 2(b)}{5 - 2}$$

$$x = \frac{0 - 2a}{3} \qquad\qquad y = \frac{0 - 2b}{3}$$

$$a = \frac{-3x}{2} \qquad\qquad b = \frac{-3y}{2}$$



subs in (1)

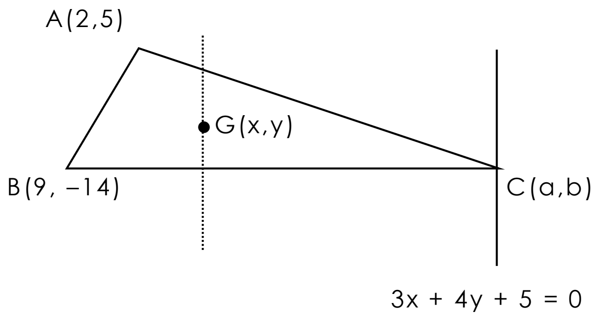
$$\left(\frac{-3x}{2}\right)^2 + \left(\frac{-3y}{2}\right)^2 + 6\left(\frac{-3x}{2}\right) - 4\left(\frac{-3y}{2}\right) + 9 = 0$$

$$\frac{9x^2}{4} + \frac{9y^2}{4} - 9x + 6y + 9 = 0$$

$$9x^2 + 9y^2 - 36x + 24y + 36 = 0 \qquad \ldots\ldots\ldots \quad \text{locus traced by P}$$

10. A(2,5) and B(9,–14) are vertices of ΔABC . The third vertex C lies on the locus whose equation is $3x + 4y + 5 = 0$. Find the locus of the centroid of triangle ABC

SOLUTION : Let G(x,y) be any point on the required locus



Since C(a,b) is any point on the locus $3x + 4y + 5 = 0$; it must satisfy the equation .

$$3a + 4b + 5 = 0 \qquad \ldots\ldots. (1)$$

G is the centroid of the ΔABC ; using centroid formula

$x = \dfrac{x_1 + x_2 + x_3}{3}$	$y = \dfrac{y_1 + y_2 + y_3}{3}$
$x = \dfrac{2 + 9 + a}{3}$	$y = \dfrac{5 - 14 + b}{3}$
$x = \dfrac{11 + a}{3}$	$y = \dfrac{-9 + b}{3}$
$a = 3x - 11$	$b = 3y + 9$

subs in (1)

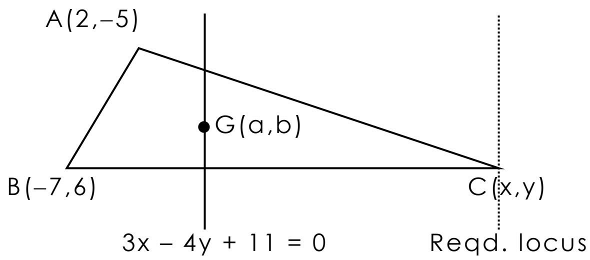
$$3(3x - 11) + 4(3y + 9) + 5 = 0$$

$$9x - 33 + 12y + 36 + 5 = 0$$

$$9x + 12y + 8 = 0 \qquad \ldots\ldots\ldots \text{ locus traced by G}$$

11. A(2, –5) and B(–7,6) are the two vertices of ΔABC . Find the equation of the locus of third vertex C , if the centroid of the triangle lies on the line $3x - 4y + 11 = 0$

SOLUTION : Let G(x,y) be any point on the required locus



Since G(a,b) is any point on the locus $3x - 4y + 11 = 0$; it must satisfy the equation .

$$3a - 4b + 11 = 0 \quad \ldots\ldots\ (1)$$

G is the centroid of the ΔABC ; using centroid formula

$x = \dfrac{x_1 + x_2 + x_3}{3}$	$y = \dfrac{y_1 + y_2 + y_3}{3}$
$a = \dfrac{2 - 7 + x}{3}$	$b = \dfrac{-5 + 6 + y}{3}$
$a = \dfrac{-5 + x}{3}$	$b = \dfrac{1 + y}{3}$
$a = \dfrac{x - 5}{3}$	$b = \dfrac{y + 1}{3}$

subs in (1)

$$3\left(\frac{x-5}{3}\right) - 4\left(\frac{y+1}{3}\right) + 11 = 0$$

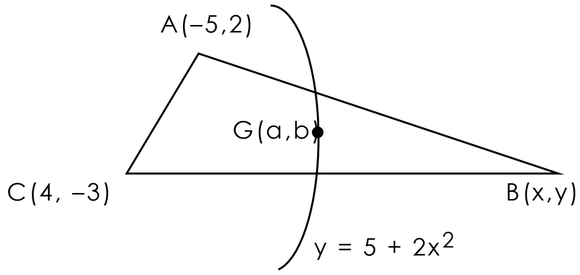
$$x - 5 - \frac{4(y+1)}{3} + 11 = 0$$

$$3x - 15 - 4y - 4 + 33 = 0$$

$$3x - 4y + 14 = 0 \quad \ldots\ldots\ldots\ \text{locus traced by C}$$

12. The centroid of a triangle always lies on the locus $y = 5 + 2x^2$. If A (–5,2) & C(4, –3) , find the equation of the locus of third vertex B

SOLUTION :



Since G(a,b) lies on the locus $y = 5 + 2x^2$; it must satisfy the equation .

$$b = 5 + 2a^2 \qquad \ldots\ldots. (1)$$

G is the centroid of the ΔABC ; using centroid formula

$$x = \frac{x_1 + x_2 + x_3}{3} \qquad y = \frac{y_1 + y_2 + y_3}{3}$$

$$a = \frac{-5 + 4 + x}{3} \qquad b = \frac{2 - 3 + y}{3}$$

$$a = \frac{-1 + x}{3} \qquad b = \frac{-1 + y}{3}$$

$$a = \frac{x - 1}{3} \qquad b = \frac{y - 1}{3}$$

subs in (1)

$$\frac{y-1}{3} = 5 + 2\left(\frac{x-1}{3}\right)^2$$

$$\frac{y-1}{3} = 5 + 2\left(\frac{x^2 - 2x + 1}{9}\right)$$

$$\frac{y-1}{3} = \frac{45 + 2x^2 - 4x + 2}{9}$$

$$y - 1 = \frac{2x^2 - 4x + 47}{3}$$

$$3y - 3 = 2x^2 - 4x + 47$$

$$3y = 2x^2 - 4x + 50 \qquad \ldots\ldots \text{ locus traced by C}$$